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Heat transfer in the corners of noncircular ducts with peripherally uniform wall heat flux

H. BARROW

Department of Mechanical Engineering, University of Liverpool, P.O. Box 147, Liverpool L69 3BX, U.K.

and

A. K. A. HASSAN

M.T.C. P.O. Box 478, Baghdad, Iraq

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INTRODUCTION

THIS PAPER has its origin in a series of numerical analyses of fully developed flow and heat transfer in ducts with triangular and cuspate cross section [1, 2]. Both laminar and turbulent flow have been studied. In these problems three peripheral thermal boundary conditions, viz. the isothermal wall, H1, uniform heat flux, H2, and conjugate heat transfer, H3, are of interest, the latter being of special importance in real situations such as, for example, heat transfer around the cladding of reactor fuel elements [3, 4]. (In all these cases, it is tacitly assumed that the axial temperature gradients are uniform corresponding to uniform heat flux in the flow direction.) The peripheral or azimuthal variation of the temperature deserves special attention in the H2 and H3 situations since its value may be a critical design parameter particularly with regard to the safety aspects. Furthermore, as pointed out by Eckert *et al.* [5], different peripheral thermal boundary conditions in very noncircular ducts result in significantly different averaged heat transfer coefficients. Of course the corners in these ducts accentuate the noncircularity, and they are the main cause of the variation of heat transfer rates around the boundary and increase the dependence of heat transfer on the thermal boundary conditions. In the numerical analyses referred to earlier, considerable success was achieved with the H1 and H3 boundary conditions, but for the H2 case, the prediction became increasingly more difficult particularly in the case of a 4-cusp duct with its very acute corner. This difficulty was attributed first to the problem of providing adequate discretization in a narrow corner in the finite difference analysis. Later however, it became apparent that a more fundamental question concerning heat transfer in an acute corner needs to be answered for the uniform heat flux condition, H2. It is evident that, as the corner is approached the interface temperature must increase monotonically to achieve the necessary potential for driving the energy into the slower moving fluid in the reduced cross sectional area of fluid. The question then arises "what is the temperature in the corner and is it determinable?".

In order to answer this question, two corner geometries, the wedge and the cusp are considered. To effect an analytical solution, the flow and heat transfer in the duct is modelled using approximations similar to those employed in tapered fin analysis [6]. How this is done is outlined in the following section where the analytical results are also presented.

THE MODEL AND THEORETICAL ANALYSIS

The corner region of the duct is modelled as shown in Fig. 1. With symmetry across the corner bisector $0X$, and assuming slug flow with velocity u in the z direction, and energy balance yields

$$y\frac{\partial^2 T}{\partial x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}\cdot\frac{\partial T}{\partial x}+\frac{q}{k}-\frac{uc\rho}{k}\cdot\frac{\partial T}{\partial z}=0. \quad (1)$$

Here c, ρ, k are the specific heat density and thermal conductivity of the fluid. As indicated in ref. [6], T is a function of x (and z) only and $\mathrm{d}s \simeq \mathrm{d}x$ which insists on a very narrow corner geometry. The last approximation allows for the uniform normal surface heat flux component to be approximated as indicated in Fig. 1. The use of slug flow does not affect the general argument. It remains to solve equation (1) for the given geometries for the temperature T and its gradient $(\mathrm{d}T/\mathrm{d}x)$. For brevity, the results only are included here

(a) Wedge see Fig. 2(a).

$$\frac{\partial T}{\partial x}=-\frac{q}{k\tan\theta}+\frac{uc\rho}{k}\cdot\frac{\partial T}{\partial z}\cdot\frac{x}{2}+\frac{C_1}{x} \quad (2)$$

$$T=-\frac{qx}{k\tan\theta}+\frac{uc\rho}{k}\cdot\frac{\partial T}{\partial z}\cdot\frac{x^2}{4}+C_1\ln x+C_2 \quad (3)$$

$(C_1, C_2 = \text{constants})$.

(b) Cusp, see Fig. 2(b)

$$\frac{\partial T}{\partial x}=\frac{1}{e}\left[-\frac{qx}{k}+C_3+\frac{uc\rho}{k}\cdot\frac{\partial T}{\partial z}\left\{rx-\frac{x}{2}\sqrt{r^2-x^2}-\frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right)\right\}\right] \quad (4)$$

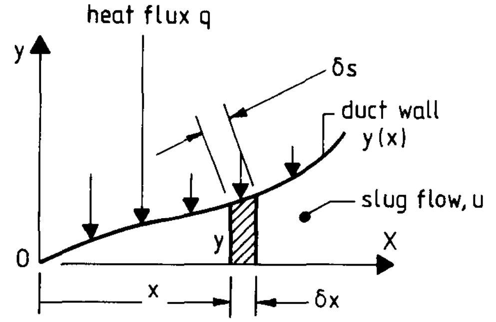

FIG. 1. The model for the corner region (tapered 'fluid fin').

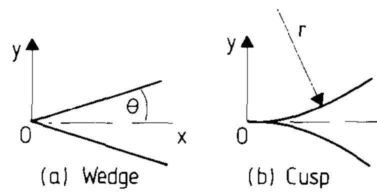

FIG. 2. Noncircular duct corner geometries: (a) wedge and (b) cusp.

$$T = -\frac{q}{k}[\sqrt{r^2-x^2}+r\ln(e)]+\int\frac{C_3}{e}\,dx + \frac{uc\rho}{k}\cdot\frac{\partial T}{\partial z}\int\cdots dx + C_4 \quad (5)$$

(where, $e = r-\sqrt{r^2-x^2}$; C_3, C_4 = constants).

Now by considering an overall energy balance for the tapered fluid fin in each geometry, it is easily shown that both C_1 and C_3 are zero.

For the cusp geometry in particular, equations (4) and (5) reveal the interesting fact that under the imposed uniform heat flux condition, H2, both the temperature and its gradient are infinite at the corner, 0, irrespective of the value of the integration constant, C_4. In equation (4), for example, l'Hôpital's rule applied to the first term on the right hand side shows an infinite value, while in the temperature solution, equation (5), the corresponding term is infinite on substituting $x = 0$. Of course, for the wedge geometry, the temperature and its gradient, equations (2) and (3) may be finite at the corner for $\theta > 0$, the results being consistent with those for the cusp geometry when θ is taken as zero. With regards to local heat transfer coefficients, the corner values are finite and zero for the wedge and cusp respectively when defined in the conventional manner.

DISCUSSION

The simple theoretical analysis of forced convection in corners with uniform energy input at the wall–fluid interface for two particular geometries has afforded an explanation of the influence of corner shape on local temperature behaviour. Furthermore, it has illustrated the reasons for the difficulty encountered earlier [1] with the numerical prediction procedure for temperatures in the neighbourhood of the corner of a cusp, for this particular thermal boundary condition. An even simpler argument for these limiting temperatures in duct corners may be made [7]. Considering the cuspate geometry, for example, for uniform heat flux through the duct surface the heat transfer in the direction x away from the corner is proportional to x. Since the area for this heat transfer is approximately proportional to x^2, it follows that the temperature gradient in the direction x is proportional to $(1/x)$. Accordingly, at the corner 0, the temperature and its gradient are both infinite (similar considerations may be used for the wedge geometry also). Of course what has been said applies to relatively small values of x or to conditions in the immediate vicinity of the corner. Previously, the answer to the question of temperature in a corner was attempted in terms of the essential balance between the normal heat flux components in the solid and fluid domains at an interface. Considering the cuspate geometry at the corner, 0, for the imposed H2 boundary condition the hypothetical situation would then arise where there would be equal and opposite components of flux at 0 in the solid domain itself. This anomaly is, of course, consistent with the predicted infinite and hence undeterminable temperature levels there. In dealing with heat transfer in noncircular ducts therefore, caution needs to be exercised if there are cuspate corners and the peripherally uniform heat flux thermal boundary condition is imposed. Numerical methods and possible theoretical solutions may predict erroneous finite temperatures and hence local heat transfer coefficients in the corners of such ducts.

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